U.G. 6th Semester Examination - 2020

MATHEMATICS

Course Code : BMTMDSHT5

Course Title : Mechanics-II

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

1. Answer any **ten** questions: 1×10=10

a) Write down the invariants of the system of forces with components X, Y, Z and couple of moment with components L, M, N.

b) Define the "stable equilibrium" of a body.

c) What is the intrinsic equation of a common catenary?

d) If T be the tension and *l* be the length of the string, when the displacement alters its length write down the work done at the end of the string.

e) Draw a sketch of a flexible string resting in a plane under the action of any given system of coplanar force in one plane.

f) What do you mean by "co-efficient of friction"?

g) Fill in the blank: In a conservative field of forces the surface of equi-pressure, equi-potential energy and equi-density _____.

h) Define the pressure at a point in a fluid.

i) What do you mean by an 'ideal fluid'?

j) How do you define a body to be a "continuous medium"?

k) Can a "rigid body" be deformable? Explain.

l) Whether the 'stress' in a continuous medium is a body force or a contact force? Explain.

m) How many elements are there, in general, in a stress matrix for a point in continuous medium?

n) Can a fluid be simultaneously 'homogeneous' and 'compressible'?

o) Define 'centre of pressure' for a plane surface immersed in a liquid at rest.

2. Answer any **five** questions: 2×5=10

a) State the principle of "Virtual Work".

b) Write down the conditions that a given system of forces should compounded into a single force.

c) Define 'Wrench' and 'Pitch' of the system of forces.

d) What is the 'Archemedian Principle'?

e) Write down the dimensions of the quantities

i) Shearing stress

ii) Density

iii) Pressure gradient

iv) Thrust

f) Write down the pressure-volume relation for a perfect gas in

i) isothermal change of state

ii) adiabatic change of state

g) What are the conditions of equilibrium for a freely floating body?

h) What is meant by the stress component τ_{xy} at a point in a continuous medium?

3. Answer any **two** questions: 5×2=10

a) Two equal uniform rods, each of weight w and length 'a' are freely jointed at A and each passes over a smooth peg at the same level. From A, a weight w′ is suspended using the equation of virtual work show that in the equilibrium position the inclination θ of the rod to the horizon is given by $\cos^3\theta = \frac{C(2w + w')}{2aw}$, C being the distance between the pegs.

b) A heavy uniform string is placed upon a rough catenary, whose axis vertical and vertex upwards, so that one extremity is at the vertex and the length of the string is equal to the parameter of the catenary; prove that the string will just rest, provided the co-efficient of friction is $\frac{2}{\pi}\log_e 2$.

c) A given mass of heterogeneous liquid is in equilibrium under external forces (X, Y, Z) per unit mass. Show that the differential equations of the curves of equal pressure and density are given by

$$\frac{dx}{\frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z}} = \frac{dy}{\frac{\partial X}{\partial z} - \frac{\partial Z}{\partial x}} = \frac{dz}{\frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y}}.$$

d) A closed right circular cylinder of height 'h' is very nearly filled with a liquid and is made to rotate about its axis which is vertical; show that if the angular velocity is $\sqrt{2gh}/a$, the thrust on the base is half the weight of the liquid when at rest where 'a' is the radius of the cylinder.

e) Explain the concept of "convective equilibrium" for the atmosphere. In which state isothermal or adiabatic, does it hold? Assuming the atmosphere to be in convective equilibrium find the expression for the absolute temperature at height z above the sea level, when the variation in gravity is neglected.

4. Answer any **one** question: $10\times1=10$

a) i) The components of the forces on the element of the fluid at (x, y, z) parallel to the co-ordinate axes are proportional to

$y^2+z^2+2\lambda yz$, $z^2+x^2+2\mu zx$,

$x^2+y^2+2\nu xy$.

Find λ, μ, ν so that the equilibrium of the fluid is possible.

ii) A plane lamina is immersed at any angle in a heavy homogeneous liquid at rest.

Show that the thrust on the lamina is a single force whose magnitude is equal to the product of the area of the lamina and the pressure at its centre of mass. Determine the co-ordinates of the point on the lamina through which the thrust passes.

b) The equal forces act along each of the straight lines

$$\frac{x\pm a\cos\theta}{a\sin\theta}=\frac{y-b\sin\theta}{\pm b\cos\theta}=\frac{z}{c}.$$

Show that their central axis must, for all values of θ, lie on the surface

$$y\left(\frac{x}{z}+\frac{z}{x}\right)=b\left(\frac{a}{c}+\frac{c}{a}\right). \qquad 10$$

OR

Two forces act, one along the line y=0, z=0 and the other along the line x=0, z=c. As the forces vary, show that the surface generated by the central axis (the axis of their equivalent wrench) is $(x^2+y^2)z=cy^2$. 10

c) i) Six equal heavy rods, freely hinged at the ends, form a regular hexagon ABCDEF,

which when hung up by the point A, is kept from altering the shape by two light rods BF and CE. Find the thrust on the rods.

ii) Show that the depth ς of the centre of pressure below the horizontal-line through the centroid is given by $\varsigma = \frac{k^2}{h}$ when the plane lamina is immersed vertically and the quantities k, h are to be explained by you.

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